Opinion Interaction Network: Opinion Dynamics in Social Networks with Heterogeneous Relationships

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ABSTRACT
Recent empirical studies have discovered that many social networks have heterogeneous relationships, which are signed and weighted relationships between individual nodes. To explore the pattern of opinion dynamics in diverse social networks with heterogeneous relationships, we set up a general agent-based simulation framework named opinion interaction network (OIN), and propose a novel model of opinion dynamics, in which the influence of agents depends on their heterogeneous relationships. Then, by conducting a series of simulations based on OIN, we find that the opinions at steady state depend on the degree of social harmoniousness and average connectivity, and the similar pattern can be observed in the network of Erdős–Rényi, small world and scale free, which illustrates that the topological properties such as short path length, high clustering, and heterogeneous degrees have few effects on opinion dynamics with heterogeneous relationships.

1. INTRODUCTION
In recent years, opinion dynamics have raised a significant attention in the field of social simulation, sociophysics and complexity science, many models have been proposed to interpret some typical phenomena, such as fragmentation, polarization, consensus, centrism and extremism[1]. Generally speaking, these models can be classified in two broad groups: discrete opinion models[2, 3, 4] and continuous opinion models[5, 6]. Meanwhile, agent-based modeling is a method of studying collective effects resulting from the interaction of many “simple” agents in a social network, and it provides an efficient and powerful tool to investigate the emerging phenomena in a social system[7]. As to different opinion dynamics models, can we set up a general framework to simulate diverse social issues? For example, social dynamics for binary opinion (e.g., voting for a election in a country, adopting a innovation in a viral market), or social dynamics for continuous opinion (e.g., evaluating the quality of food within a community). In this paper, we propose a general agent-based simulation framework named opinion interaction network (OIN), which can not only support both discrete and continuous models of opinion dynamics, but also can simulate the process of opinion evolution in heterogeneous complex networks, so it lays the experimental foundation for studying opinion dynamics in different complex networks with heterogenous relationships.

Most models of opinion dynamics consider the relationships of people are homogeneous, and opinion interactions take place within neighbors who are connected by simple links in a social network[3, 4, 5, 6]. Nevertheless, many real social networks display a large heterogeneity in the capacity and the intensity of the connections, which can be described in terms of weighted networks[8]. Moreover, another important property of relationships is that connections of in-
individuals often reflect a mixture of positive (friendly) and negative (antagonistic) interactions, which can be mapped into signed links in a network. Many realistic online social networks have been discovered the existence of positive and negative relationships, such as online rating sites “Epinions”, online discussion sites “Slashdot”, and Wikipedia[8]. As what Zedong Mao in China said “We should oppose what enemies support, and support what enemies oppose”, no matter what people discuss, relationships can play a key role in opinion change. Therefore, the heterogeneity of relationships (weighted and signed properties) should be taken into account in modeling opinion dynamics. In this paper, we consider the heterogeneous relationships as the driving force of opinion drift when a pair of agents interact, and propose a novel opinion dynamics model consisting both of attractive and repulsive effects, where the influence is dependant on the heterogenous relationships.

Meanwhile, lots of literatures focus on the influence of network structures to opinion dynamics, which are all on the hypothesis that the relationships of networks are homogeneous. Song et al[10] investigate the evolution of Snajdz model on several complex networks, and find a phase transition from the state with no consensus to the state with complete consensus. Suchecki et al[11] study the voter model evolves on different classes of complex networks, and find how different features of complex networks (dimensionality, order, heterogeneity of the degree) impact the ordering dynamics of the voter model. In particular, studies on Defuant model with different social networks have been paid much attention. Stauffer and Meyer-Ortmanns[12] study Defuant model on a scale free network, and find that number of different final opinions is proportional to the number of people. Moreover, The Defuant model on a directed scale free network exhibits different properties for confidence parameter compared with the undirected case[13]. Guo and Cai[14] study opinion dynamics of improved Defuant model on small world network and scale free network, and find the impact of the confidence parameter and complex network topologies in opinion dynamics. Kurmyshev et al[7] propose a mixed model including two psychological types of agents, which are concord agents and partial antagonism agents. They also study the dynamics and bifurcation patterns of opinion interacting in different social networks, and find that group opinion formation is almost independent of the topology of networks. As to the heterogeneous relationships which are more general and realistic in a society, the question we address is how opinions evolve with diverse network topologies?

We conduct a series of simulations of opinion dynamics in signed networks based on OIN, and our major findings are as follows: The opinions at steady state undergo a transition from the bipolarization phase to the consensus phase as increasing the parameter \( h \), which denotes the ratio of the number of positive relationships to the total. Meanwhile, the bifurcation of opinions is independent to the scale of population in Erdős-Rényi network, but is strong related to the average connectivity \( k \). It also exhibits the pattern that the larger \( k \) is, the more dispersive the final opinions are, except when the opinions collapse into consensus at \( h = 1 \). Interestingly, the pattern of opinion dynamics is similar in the network of Erdős-Rényi, small world and scale free, meaning that the topological properties of diverse networks such as short path length, high clustering, and heterogeneous degrees have few effects on opinion dynamics with heterogeneous relationships.

The work is organized as follows. In Section 2, we briefly describe the simulating framework OIN for opinion dynamics. In Section 3, we propose a novel opinion dynamics model with heterogeneous relationships. A series of simulations based on OIN are presented in Section 4. Finally, conclusion and discussion are given in Section 5.

### 2. FRAMEWORK OF OIN

To explore opinion interaction and evolution in social systems with diverse topologies, we set up a simulating framework named OIN, which can be used to observe what patterns of phase transition at the macro-level with different topologies when agents at the micro-level interact and change opinions in accordance with a fixed principle. Opinion interaction network specification is described as OIN\((O, G(V,E,W))\), Rule\((O)\), \(N\), \(T\), and the detail interpretation of elements are illustrated as table 1.

As we can see, the elements \(N\) and \(T\) are control parameters of system, which are related to the scale and precision of a simulation. In fact, the smaller threshold \(T\) we consider, the more slowly the system converges. In our study, we use an empirical value and set \(T = 10^{-20}\) to determine whether system reaches the steady state. Then we mainly focus on the other elements in OIN.

**Opinion space** \(O\) In the field of opinion dynamics, opinion space \(O\) is a set of variables indicating the possible position or position interval for each individual on a special topic. It can be divided into two categories: discrete opinion space and continuous opinion space, which can be used to model different social issues ranging from politics to parenting, from art to zoology. In principle, compared with discrete opinion space, continuous opinion space can provide more details, for example, the feeling on a film, the trends towards voting, the attitudes for a debate, etc. Let \(o_i\) denotes the opinion of agent \(i\) in OIN, \(o_i \in \{0, 1\}\) or \(o_i \in (-1,1)\) represents each agent has to adopt one of two options, which can be used in voter model, majority model and Ising spin model[15]. In contrast, \(o_i \in [0,1]\) or \(o_i \in [-1,1]\) implies the opinion varies from opposition to support smoothly over a topic, which can be used in Defuant model[5], H-K model[6] and social judgement model[1]. In our study, we choose the continuous opinion space to model the opinion of individuals, and the detail can be seen in the next section.

**Social network** \(G(V,E,W)\) We consider a society envisaged as a social network denoted by \(G(V,E,W)\), in which agents can communicate and exchange opinion within neighborhood. When \(w = 1\), the relationships are homogeneous in a society. In contrast, relationships are heterogenous in terms of different \(w\). To emphasize the effects caused by heterogeneous relationships including both friendship and hostility, we assume \(w\) is independently drawn from the uniform distribution \(U(-1,1)\). We also define a parameter \(0 \leq h \leq 1\)

<table>
<thead>
<tr>
<th>Table 1: Specification of OIN</th>
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<tr>
<td><strong>Element</strong></td>
</tr>
<tr>
<td>(O)</td>
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<tr>
<td>(G(V,E,W))</td>
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<tr>
<td>Rule((O))</td>
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<tr>
<td>(N)</td>
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<td>(T)</td>
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**Rules governing the opinion interaction**

Two psychological types of agents are considered in OIN, which can be used to observe what patterns of phase transition at the macro-level with different topologies when agents at the micro-level interact and change opinions in accordance with a fixed principle. Opinion interaction network specification is described as OIN\((O, G(V,E,W))\), Rule\((O)\), \(N\), \(T\), and the detail interpretation of elements are illustrated as table 1.
to describe the harmoniousness of society, which can be considered as the ratio of positive relationships in a society.

The topologies of social networks are commonly used in OIN including complete network, Erdős-Rényi random graph[16], Regular lattice[17], small world network[17], and scale free network[18]. In practice, lots of empirical measurement works have found that social networks have the properties of small world and scale free, such as online social network (Flickr[19], Blog[20], Twitter[21, 22], YouTube[23] and etc), co-authorship network[24, 25] and P2P social network[26]. In our study, we mainly focus on the three complex networks:

- **Erdős-Rényi random graph** is proposed by Erdős and Rényi with the original purpose of studying, and it is constructed by connecting nodes randomly with equal probability. The degrees of random graph follow Poisson distribution. Due to its simplicity, it is always used to investigate the dynamics on the graph.

- **Small world network** has the properties of small average distance and high clustering coefficient, which can be generated by Watts and Strogatz (WS) model. This model is defined on a ring over $N$ nodes. Initially each node in the ring is connected its $k$ neighbors, and then shortcuts are created by rewiring links randomly with probability $p$, especially the network is named Regular lattice when $p = 0$. Therefore, the small world model can be considered as a homogeneous network, in which all nodes have almost the same degrees.

- **Scale free network** has the properties of power-law degree distribution, which can be generated by Barabasi-Albert (BA) model. This model is grown by attaching new nodes each with $m$ edges that are preferential attached to existing nodes with high degree, and it can generate a network of which the degree distribution follows a power law with an exponent $\alpha \sim 3$. Compared with small world network, it is a heterogeneous network which exists a few hubs with very large degrees.

**Interaction rule** 

*Rule(O)* In opinion dynamics, interaction rule describes when and how opinions interact and change between individuals, and it is related to the opinion space $O$. Accordingly, interaction rule *Rule(O)* can be considered as the kernel of OIN. Due to the sparse data for realistic interactions, it is difficult to obtain an accurate interaction rule for opinion dynamics. However, some research results of sociology and psychology can give us a guidance to model the behavior of interactions. Within the framework of continuous opinion space, there exists two important concepts: bounded confidence[15] and social judgement theory[1]. The first concept considers that compromise of opinions takes place when the opinions are sufficiently close to each other, while the latter takes both of attractive and repulsive effects into account, and the drift of opinion also depends on the distance of opinions. On the basis of above concepts, three typical interaction rules have been proposed and they are described in detail as follows:

- **Defuquant model** is proposed under the concept of bounded confidence. At each time step, a pair of agents $i$ and $j$ are chosen randomly from $G$ where the edge $e_{ij} = 1$, then the agents update their opinions as follows:

  \[
  \begin{aligned}
  o'_{ij} &= o_i + \mu \cdot (o_j - o_i), \\
  o'_{ji} &= o_j + \mu \cdot (o_i - o_j),
  \end{aligned}
  \]  

  if $|o_i - o_j| < \epsilon$.  

  where $\mu$ is the convergence parameter and $\epsilon$ represents the confidence of an agent.

- **HK model** is similar to Defuquant model, which is also based upon bounded confidence. The significant differences between two models are interacting regime and updating rule. At each time step, each agent interacts with his neighbors and updates the opinion synchronously. Taking an agent $i$ for example, the updating rule is defined:

  \[
  o'_i = \sum_{|o_i - o_j| < \epsilon} \frac{o_j}{N_i}.
  \]  

where $N_i$ means the number of neighbors who satisfy the condition $|o_i - o_j| < \epsilon$.

- **Social judgement model** is somewhat different to Defuant model, and it assumes opinion dynamics should have both attractive and repulsive effects, which depends on the differences of opinions between a pair of nodes. Thus, this model introduces a new parameter $t$ to denote the threshold of rejection, and the updating rule is defined:

  \[
  \begin{aligned}
  o'_i &= o_i + \mu \cdot (o_j - o_i), \\
  o'_j &= o_j + \mu \cdot (o_i - o_j),
  \end{aligned}
  \]  

if $|o_i - o_j| < \epsilon$,  

if $|o_i - o_j| > t$.  

As we can see, the above models are defined on the hypothesis of homogeneous relationships, and they all use threshold parameters to decide whether opinion interaction takes place. However, the threshold parameters have no actual meaning and implication on the outcome. Therefore, as to the heterogenous relationships, we should keep the attractive and repulsive effects reserved, and measure the opinion drift by their heterogeneous relationships when a pair of individuals interact, which is more practical and intuitive than threshold parameters. The detail of opinion dynamics model with heterogenous relationships is described in the next section.

3.  **OPINION DYNAMICS MODEL WITH HETEROGENOUS RELATIONSHIPS**

Consider a signed social network $G(V, E, W)$ with heterogenous relationships where $w \in [-1, 1]$, and the opinion space $O$ is continuous on the interval $[-1, 1]$. At each time step, an agent $i$ is chosen randomly from $G$ and interacts with his neighbors $N_i = \{j | e_{ij} \in E, j \in V\}$ sequentially, until all neighbors have interacted with the target agent $i$. We define the interaction rule *Rule(O)* between a pair of agents $i$ and $j$ as follows:

  \[
  o'_i = o_i + f(w_{ij}, o_i, o_j) \ast (o_j - o_i).
  \]  

where $f(w_{ij}, o_i, o_j)$ is defined as follows:

  \[
  f(w_{ij}, o_i, o_j) = \begin{cases} 
  w_{ij}, & \text{if } w_{ij} \geq 0, \\
  w_{ij} \frac{1 - o_i}{1 - o_j}, & \text{if } w_{ij} < 0 \text{ and } o_i \geq o_j, \\
  w_{ij} \frac{1 - o_j}{1 - o_i}, & \text{if } w_{ij} < 0 \text{ and } o_i < o_j.
  \end{cases}
  \]  

The general idea behind the updated rule is that: each opinion moves into the direction of the other for a pair of friends, while each opinion moves into an opposite direction for a pair of antagonists, and its moving amount is defined as opinion distance $|o_j - o_i|$ multiplied by the relative drifting function $f(w_{ij}, o_i, o_j)$. 

Figure 1: Opinion interaction rule with heterogeneous relationships. a) $w_{ij} \geq 0$ b) $w_{ij} < 0$ and $o_i \geq o_j$.

Suppose target agent $i$ and agent $j$ is a pair of friends ($w_{ij} \geq 0$) (as shown in Fig. 1(a)), the movement of opinion for the target agent $o'_i - o_i$ is proportional to the opinion distance $d_0 = |o_j - o_i|$ and their relationship $f(w_{ij}, o_i, o_j) = w_{ij}$, which is similar to the Deffuant model. As to the antagonistic relationship ($w_{ij} < 0$), the interaction dynamics become more complex. Without loss of generality we assume that $o_i \geq o_j$ (as shown in Fig. 1(b)). For the target agent $i$, the maximal distances to the both of extreme values are $d_1 = 1 + o_i$ and $d_2 = 1 - o_i$, and the opinion moving direction is towards extreme value 1. Thus, we define the relative drifting function as $f = w_{ij} \frac{d_2}{d_1} = w_{ij} \frac{1 - o_i}{1 + o_i}$, which can not only ensure the moving distance is related to the relationship, but also guarantee the updated opinion $o'_i$ is in the interval $[-1, 1]$.

Note that different interaction sequences will lead to different results if an agent has multiple neighbors. For the sake of simplicity, we select sequential interaction process by descending $w$. That is, the target agent interacts with the most intimate friend (the neighbor with the biggest $w$) first, then with the second one, and repeat this process until all his neighbors communicate with him.

In order to explore the impacts of heterogeneous relationships and social networks on opinion dynamics, we use OIN framework to simulate opinion evolution for a social system, and the parameters of social system include:

- Parameter $h$ regulates the degree of harmoniousness in a society, which is the fraction of positive relationships in a society.

- Parameter $Para(G)$ describes the topological characteristics of social network $G$. Namely they are average connectivity $k$ in a complex network and rewiring probability $p$ in WS model, which determines some important statistical measures such as cluster coefficient and average path length.

4. NUMERICAL RESULTS BASED ON OIN

We study opinion dynamics with heterogenous relationships by means of Monte Carlo simulations based on OIN, and the results represent an average over 100 realizations for each configuration. If not specifically mentioned, The distribution of opinions here represents the density distribution of opinions when the social system reaches steady state.

First of all, we pay most of our attention to opinion dynamics in Erdős-Rényi random networks, which is simple to analyze and understand the effects of social harmoniousness parameter $h$ on opinion dynamics. Fig. 2 shows the distribution of opinions as a function of parameter $h$ on a random network. We notice that the phases of social system exhibit a strong dependence on the harmoniousness parameter $h$, and they undergo a transition from the bipolarization state to the consensus state as increasing $h$. For small $h$ ($h < 0.5$), opinions of agents reaches a state of bipolarization, and most of agents can be approximately divided into two extreme groups with equal size. As increasing the ratio of the number of positive relationships, opinions of agents shifts towards pluralism, which means diversity of opinions emerge and the distribution of opinions becomes more uniformly than bipolarization state. Then, the opinions of agents have a tendency to group together, and they collapse into consensus when $h = 1$ finally, which can be also found in opinion dynamics of Deffuant model[5]. Above phenomenon illustrates that the presence of negative relationships can speed the formation of opinion divergence, and the social system can be divided into two extreme groups when the degree of harmoniousness is relative small.

Figure 2: Distribution of opinions with increasing $h$ in Erdős-Rényi random graph. The parameters of OIN are: $N = 100$ and $k = 15$. The x-axis represents the harmoniousness parameter $h$, and y-axis is the density distribution of opinions.

To capture the characteristics of opinion dynamics, we employ the variance measure to express the dispersion of the opinions like[27]. If opinions of agents reach consensus state, the corresponding variance is 0. In contrast, the variance is 1 if they are complete bipolarized (half of the agents have opinion -1 and the other half 1). Naturally, the variance varies between 0 and 1, and it is 1/3 if the distribution of opinions is uniform. Thus, the variance measure is smaller than 1/3 if opinions have a tendency to consensus, and bigger than 1/3 if opinions have a tendency to polarize. Fig. 3 shows the variance measure as a function of $h$ with different number of population $N$ and average connectivity $k$. Along with the results in Fig. 2, we observe that the variance measure decreases from the value greater than 0.9 (bipolarization) to 0 (consensus) with different settings. Furthermore, compared with different $N$ and $k$, we can find that the distribution of opinions is independent to the scale of population, but is strong related to the average connectivity. Meanwhile, we also observe that the variance trajectory with larger $k$ is always above than the one with
smaller $k$ except when $h = 0$, meaning that the number of people who participate in a discussion is very important to the final result, and the larger the average connectivity is, the more dispersive the final opinions are. As to a complete harmonious society ($h = 1$), no matter what the average connectivity is, all opinions of agents will converge to the value near zero.

Figure 3: Variance measure as a function of $h$ in Erdős–Rényi random graph. The parameters of OIN are: $N = 50, 100, 200$, $k = 10, 15, 20$.

Then, we focus on opinion dynamics in regular lattice, which is considered as a network with the randomness probability $p = 0$ in WS model. The pattern of opinion dynamics is similar to the case in Erdős–Rényi random networks (as shown in Fig. 4), and the phase transitions are also dependent on the harmoniousness of society $h$.

In a realistic society, opinion polarization is considered as a particularly relevant correlate of social conflict and unrest[28]. Therefore, it is meaningful to prevent the emergence of opinion polarization in most situations. To explore how extreme opinions vary with different parameters in detail, we define another measure $P_o$ to describe the probability of extreme opinions, and it is:

$$P_o = \text{Prob}\{o \leq 1 - \sigma\} + \text{Prob}\{o \leq -1 + \sigma\}.$$  

(6)

where $\sigma$ represents the boundary of extreme opinions we considered, and let $\sigma = 0.1$ in our study. Fig. 5 shows $P_o$ as a function of $h$ with different $k$. We observe that the probability of extreme opinions with the larger $k$ is always bigger than the one with the smaller $k$, in condition of the same $h$, and it also has a decline trend as increasing $h$. The phenomenon illustrates that there are two strategies we can adopt to reduce polarization phenomenon: promoting social harmony and limiting the average number of neighbors for each individual in a social network. Obviously, the first strategy can be considered as the operation of increasing $h$, while the other one represents the operation of reducing $k$. For example, consider a social system with the average connectivity $k = 32$ is full of negative relationships ($h = 0$), the individuals of extreme opinions take up about 92% of population after opinion interactions. To reduce the polarization of the social system, one of feasible methods is to promote the social harmony, and the probability of extreme opinions reduces to about 75% when $h = 0.8$. To obtain the effect as well as previous operation, we can also reduce the average connectivity of the interaction network, and the similar result can be observed in Fig. 5 when $k$ reduces to 4.

Figure 4: Distribution of opinions in a) Erdős–Rényi random network, and b) regular lattice network. The parameters of OIN are: $N = 100$ and $k = 10$.

Besides the parameter of average connectivity $k$, WS model is ruled by another parameter, that is the randomness probability $p$. It determines the properties of the average path length and cluster coefficient in small world networks. To measure the effects of $p$ to opinion dynamics, we conduct a series of simulations with different configurations based on OIN, and the configurations include:

1. $p = [0, 0.1, 0.2...0.9, 1]$, $k = 4$ and $h = [0, 0.5, 0.8, 1]$;
2. $p = [0, 0.1, 0.2...0.9, 1]$, $k = 8$ and $h = [0, 0.5, 0.8, 1]$;
3. $p = [0, 0.1, 0.2...0.9, 1]$, $k = 10$ and $h = [0, 0.5, 0.8, 1]$;
4. $p = [0, 0.1, 0.2...0.9, 1]$, $k = 12$ and $h = [0, 0.5, 0.8, 1]$;
5. $p = [0, 0.1, 0.2...0.9, 1]$, $k = 16$ and $h = [0, 0.5, 0.8, 1]$;
6. $p = [0, 0.1, 0.2...0.9, 1]$, $k = 32$ and $h = [0, 0.5, 0.8, 1]$.

We first compute the probability density of opinions $Q$ for each configuration. Then, fixing the parameter $k$ and $h$, we use the Pearson correlation coefficients to measure the pairwise similarity of $Q$ for a pair of $p$, and its corresponding correlation matrix is denoted by $R_{v, k} = (\rho_{i,j})_{k \times k}$, where $i, j \in p$, thus we have $|k| \times |h| = 6 \times 4 = 24$ correlation matrixes for
Figure 5: $P_\sigma$ as a function of $h$ in regular lattice where $\sigma = 0.1$. The parameters of OIN are: $N = 100$ and $k = 4, 10, 16, 32$.

Figure 6: a) Distribution of opinions in scale free network. The parameters of OIN are: $N = 100$ and $k = 9.5$; b) $P_\sigma$ compared with scale free network (SFN) and small world network (SWN).

5. CONCLUSION

Opinion dynamics on diverse networks with homogeneous relationships have attracted considerable attentions in the last years. In this paper, with the purpose of exploring the pattern of opinion dynamics in different complex networks with signed and weighted relationships, we set up a general simulation framework OIN and propose a model of opinion dynamics, which consider the heterogenous relationships as the driving force of opinion drift. By conducting a series of simulations in Erdős-Rényi, small world and scale free network, our major findings are as follows: The opinions at steady state undergo a transition from the bipolarization phase to the consensus phase as increasing $h$. Meanwhile, the distribution of opinions is independent to the scale of population, but is strongly related to the average connectivity $k$. It also exhibits the pattern that the larger $k$ is, the more dispersive the final opinions are, except when the opinions collapse into consensus at $h = 1$. Interestingly, the pattern of opinion dynamics with the same $k$ is similar in the network of Erdős-Rényi, small world and scale free, which means that the topological properties of complex networks have few effects on the opinion dynamics.

As it has been seen, we find some interesting results based on the model taking heterogeneous relationships into account. However, in realistic individuals interactions, signs and weights of relationships can not keep constant, but change with the process of opinion dynamics. Therefore, we plan to investigate an adaptive coevolutionary network model coupling weights of relationships and opinion dynamics, in order to better understand the relation of opinion dynamics and complex networks.

6. ACKNOWLEDGMENTS

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Table 2: Pairwise Correlation Coefficients of Opinions Distribution for Diverse $p$.

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7. REFERENCES


